**2.5**

**Rational Zeros Theorem, Finding Rational Zeros,**

**Upper & Lower Bounds, Descartes’s Rule of Signs**

**Rational Zeros Theorem**: Real zeros of polynomial functions are either rational zeros (rational numbers) or irrational zeros (irrational numbers).

Suppose *f* is a polynomial function of degree *n* 1 of the form

*f(x) =anxn + a n-1 xn-1 + … a1x + a0,*

with every coefficient an integer and *a0,*0. If *x* = *p/q* is a rational zero of *f,* where *p* and *q* have no common integer factors other than 1, then

* *p* is an integer factor of the constant coefficient *a0,* and
* *q* is an integer factor of the leading coefficient *an*.

Example:

Find all the possible rational zeros of *f(x) = 3x3 + 4x2 - 5x - 2*.

**Potential** rational zeros: 

Now how do we find **all** rational zeros of the polynomial?

**Upper and Lower bound tests for Real Zeros:**

Let *f* be a polynomial function of degree *n* ≥ 1 with a positive leading coefficient. Suppose *f(x)* is divided by *x - k* using synthetic division.

* If *k* ≥ 0 and every number in the last line is nonnegative (positive or zero), then *k* is an **upper bound** for the real zeros of *f.*
* If *k* ≤ 0 and the numbers in the last line are alternately nonnegative and nonpositive, then *k* is a **lower bound** for the real zeros of *f*.

Example:

Prove that all of the real zeros *of f(x) = 2x4 - 7x3- 8x 2+ 14x + 8* lie in the interval [-2,5].

(Prove that -2 is a lower bound and 5 is an upper bound using synthetic division.)

The French mathematician Renè Descartes (1596-1650) recognized a connection between the roots of a polynomial equation and the + and – signs of the standard form.

**Descartes’s Rule of Signs:**

Let P(x) be a polynomial with real coefficients written in standard form.

* The number of positive real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(x) or is less than that by an even number.
* The number of negative real roots of P(x) = 0 is either equal to the number of sign changes between consecutive coefficients of P(-x) or is less than that by an even number.

In both cases, count multiple roots according to their multiplicity.

**Example using Descartes’s Rule of Signs:**

What does Descartes’s Rule of Signs tell you about the real roots of



Answer:

